Geometer's Sketchpad Assignment (extra credit)
Math 430

Geometer's sketchpad is a useful program for learning and teaching elementary geometry. It allows you to draw dynamic diagrams of triangles, circles, angle bisectors, perpendicular lines, and more. Geometer's sketchpad (version 5) is available on all the computers in the math department, but if you happen to have an older version elsewhere, that should be fine. This assignment is designed to help you learn the basic features of GSP and review some basic Euclidean geometry. As you will see in this assignment, sometimes drawing diagrams in GSP can lead you to discover Theorems. One can also use GSP as a tool to teach geometry.

To turn in this assignment: email it to me as an attached GSP file by Tuesday, December 11. Please name your file yourlastname.gsp when you send it to me.

GSP is easy to learn by experimenting with the different tools and menus built-in. Detailled instructions can be found in the first few chapters of Venema's Exploring Advanced Euclidean Geometry with Geometer's Sketchpad. Many of the problems in this assignment are taken from this Venema guide or assignments written by Aaron Magid, Karin Melnick and Frances Gulick in past semesters.

Problem 1. Recall the false proof that all triangles are isosceles in Problem 1 of HW1. Draw a diagram illustrating the proposed proof, and then explain why the proof fails.

First draw a triangle. Label the vertices A, B, C. (To label something, select it and then go to the Display menu.) Draw the angle bisector at A. (To construct things such as angle bisectors, select the vertex and two sides and go to the Construct menu.) Draw the perpendicular bisector to BC. Label the point of intersection D. Draw perpendicular lines through D to the sides AB and AC. Note you may need to extend the lines AB and AC. Label the points of intersection E and F respectively as in the homework problem. Explain why the "proof" given in the homework assignment 1 fails (you should include this as a textbox in your assignment).

Note that if you follow this construction, you can move the vertices A, B, and C around in the plane to explore the possibilities for all triangles, including what happens when $\triangle ABC$ is actually isosceles.

Problem 2. Create a new page for each part of this problem. These should all save as a single file with multiple pages. (Go to File - Document Options - add page.)

(a) Construct a triangle. Label its vertices A, B, and C. Draw the perpendicular bisectors of all three sides. Do they intersect in a single point as you drag the three vertices around? Label the point of intersection P. This is called the circumcenter of the triangle. Draw the circle centered at P containing the vertex A of the triangle. What do you notice about the circle?

(b) Do the same thing with the altitudes of $\triangle ABC$ instead of the perpendicular bisectors. Convince yourself that these three lines intersect in a point O which we will call the orthocenter.

(c) Do the same thing with the angle bisectors of $\triangle ABC$. Convince yourself they always intersect in a single point. Label this point I. This is called the incenter or center the inscribed circle.

(d) Do the same thing with the three medians of the triangle. A median is a line that goes through a vertex and the midpoint of the opposite edge. Label this point E for centroid. (This is the "center of mass" of the triangle.)

(e) Create a new page by duplicating page 2(d) (choose File: Document Options: Add Page: Duplicate Page and then choose 2d from the list). Select the 3 medians and hide them (choose the lines but be careful that the point E is not selected, and then choose Display: Hide Lines). Create the other three points, P, O, I and then hide the perpendicular bisectors, altitudes and angle bisectors. As you move the vertices of the triangle what do you notice about these four points? Draw a line segment from O to P. The line through O and P is called the Euler line; sometimes the segment is also given that name. What happens to the line segment as you move the vertices of the triangle? How many of the four points lie on the Euler line? Which of the four points can lie outside the triangle? When do they lie outside the triangle? For what kind(s) of triangles are there fewer than four distinct points?

Problem 3. **(a)** Again, make a new page in the same file. Construct a quadrilateral. Label three of the vertices X, Y, Z. Now label the midpoints of each of the edges A, B, C, D. Form a new quadrilateral by joining A, B, C, and D by edges (in the same order as the edges on the original quadrilateral). Go to the Display menu and change the color of the new quadrilateral to be different than the original. Now select the vertices X, Y, Z and go to the Display menu and click Animate. Isn't that fun? No matter what X, Y, Z are, the midpoint quadrilateral $ABCD$ always has a special property. Can you make a conjecture as to what this property is?

(b) We will prove something about $ABCD$ when we get to Chapter 4, but in the meantime, let's use GSP to measure things. Stop the animation, and draw a line that goes through AB and CD (should be opposite edges of the quadrilateral $ABCD$). Now measure the alternate interior angles. You can do this by selecting three vertices which define the angle, and then going to the Measure menu. Assuming for a moment that the measurements are exact, what does this prove about the sides AB and CD?

Now do the same thing for the other pair of opposite sides AD and BC. Draw a line through them, and then measure alternate interior angles. Move your line around. Both measurements should change, but what do you notice about the way that the two measurements change?