Geometry, Fall 2012

University of Maryland, Department of Mathematics course 430

HW 2: (due by September 20, in class.)

Please write-up your own solutions to problems in an organized and neat fashion. If collaborating in the problem solving process please write the names of the people with whom you collaborated next to each solution.

0. Let A, B, C be three non-colinear points in the plane. Using Euclidean geometry, construct the center O of the circle. You may use anything in Book I of Euclids Elements. As with any construction, be sure to find the desired point O and prove that it is indeed the center of the circle.

1. Let $\mathbb{C}$ denote the complex numbers, and let $\mathbb{R} \subset \mathbb{C}$ denote the real numbers. Let $z_1, z_2, z_3, z_4 \in \mathbb{C}$.
(i) Assume that no three of these points are colinear. Prove that these four points lie on a circle if and only if

$$\frac{z_1 - z_2}{z_2 - z_3} \cdot \frac{z_2 - z_4}{z_1 - z_4} \in \mathbb{R}.$$

(ii) In class we proved that $z_1, z_2, z_3 \in \mathbb{C}$ are colinear if and only if $\frac{z_1 - z_3}{z_2 - z_3} \in \mathbb{R}$. *Without using this fact,* prove directly that $\frac{z_1 - z_3}{z_2 - z_3} \in \mathbb{R}$ if and only if $\frac{z_2 - z_1}{z_3 - z_1} \in \mathbb{R}$.

2. Let $\mathbb{Q}$ denote the rational numbers. Assume that $A = (x_1, y_1), B = (x_2, y_2), C = (x_3, y_3)$ are points in the plane with $x_i, y_i \in \mathbb{Q}, i = 1, 2, 3$. Let $O = (x_4, y_4)$ denote the center of the circle circumscribing the triangle ABC. Prove that $x_4, y_4 \in \mathbb{Q}$.

3. Stillwell (Four Pillars of Geometry), problems 2.7.1, 2.7.2, 2.7.3, 2.7.4.

4. Stillwell, problems 2.8.1, 2.8.2, 2.8.3.