

Geometry, Fall 2012

University of Maryland, Department of Mathematics course 430

HW6: (due in class October 25)

Please write-up your own solutions to problems in an organized and neat fashion and staple your sheets. If collaborating in the problem solving process please write the names of the people with whom you collaborated next to each solution.

0. *Reflection is the limit of inversions:*

Let C_R denote a circle of radius R centered at $(-R + 1, 0)$, so that $(1, 0) \in C_R$ for each $R > 0$. Let $I_R : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ denote inversion about C_R . Let $\bar{r}_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ denote reflection about the line $x = 1$. Prove that $\lim_{R \rightarrow \infty} I_R = \bar{r}_1$. (In other words, prove that for each $(x, y) \in \mathbb{R}^2$ holds $\lim_{R \rightarrow \infty} I_R(x, y) = \bar{r}_1(x, y)$.)

1. (i) GOS, Problem 3.5.2.

(ii) Let $\text{Iso}(S^2)$ denote the group of isometries of S^2 . Prove that the subset of $\text{Iso}(S^2)$

$$\{f \in \text{Iso}(S^2) : f \text{ is a rotation}\}$$

is a subgroup.

2. Stillwell (GOS), Problems 3.5.3, 3.5.4, 3.5.5.