Differential Geometry I, Autumn 2013

University of Maryland, Department of Mathematics course 436

HW 2: (due by October 15, in class.)

Please write-up your own solutions to problems in an organized and neat fashion. Collaborating in the problem solving process is not allowed for this homework.

1. Let M denote the torus of Callahan's Problem 5(e) (p. 219):

(a) Exhibit explicitly coordinate charts for M.

(b) Choose two charts that intersect and write down explicitly the transition maps between these two charts as well as their inverses. (Do not need to do this for all charts, just two.)

(c) Write down the differential of the transition maps and their inverses (just between the two charts of (b)).

2. Let (M, g) be a Riemannian manifold. Denote by dV_g the Riemannian volume form, defined as a section of $\Lambda^n TM$ with the property that $dV_g(E_1, \ldots, E_n) = \pm 1$ whenever $E_i \in T_pM, i = 1, \ldots, n$ is an orthonormal basis of T_pM with respect to g (i.e., $g(E_i, E_j)(p) = \delta_{ij}$). Show that with respect to local coordinates $x_1, \ldots, x_n$ valid near p,

$$dV_g(x) = \sqrt{\det[g_{ij}(x)]}dx^1 \wedge \cdots \wedge dx^n.$$