Kähler manifolds, Autumn 2014

University of Maryland, Department of Mathematics course 868K

HW 5:

1.
Prove or disprove the following claims related to the proof of the Kähler identities:
A) $\iota_k \bar{e_l} + \bar{e_l} \iota_k = 0$.
B) $\iota_k e_l + e_l \iota_k = 0$.

2.
Show that $[\Lambda, L] \circ \Pi^{p,q} = (n - p - q)\Pi^{p,q}$.

3.
Let M be a projective surface and let C be a -1-curve in M. Denote by π the map furnished by the Castelnuovo–Enriques Theorem. Give a detailed proof that:
A) $\pi(C)$ is a point.
B) $\pi(M)$ is smooth at $\pi(C)$.

4.
Follow through the steps of the proof of the Castelnuovo–Enriques theorem for M being the blowup of $\mathbb{P}^2$ at one point. Namely, use the explicit description of the blowup to embed this manifold in a projective space, identify the -1-curve, and then construct the map π and prove that its image is $\mathbb{P}^2$.